Prof. Hadley

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THE

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MARCH, 1861.

EDITED BY

J. D. RUNKLE, A.M., A.A.S.

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THE MATHEMATICAL MONTHLY.

The excellent notices which the Monthly continues to receive from both sides of the Atlantic must be a source of great encouragement to all its friends. The *Lady's and Gentleman's Diary* for 1861, edited by the eminent mathematician, PROF. W. S. B. WOOLHOUSE, contains the following: —

"This American periodical, which has now completed its second volume, continues to be most ably conducted, and is well supplied by talented correspondents. Besides numerous mathematical questions and solutions, it comprises original papers of considerable variety and sterling interest, and its publication must contribute largely to the diffusion and advancement of mathematical science."

A recent number of the *Boston Daily Transcript* gives the following item: —

"AMERICAN LITERATURE ABROAD. — Fourteen columns of the last Athenæum are devoted to two American authors — Motley and Emerson. That paper, which rarely commends anything from this side of the Atlantic, gives 'The Mathematical Monthly' the following first-rate notice. That magazine is published at Cambridge, and edited by Mr. J. D. Runkle: —

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Those in arrears are earnestly requested to remit their subscriptions, if possible, before the issue of the next number.

SEVER & FRANCIS, PUBLISHERS,

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THE

MATHEMATICAL MONTHLY.

Vol. III. . . . MARCH, 1861. . . . No. VI.

PRIZE PROBLEMS FOR STUDENTS.

I. Given

$$(1) \quad a^2 - 3x^4 + (2+4y)x^3 + (4y+1-2a)x^2 - (2a+4ay)x = 4y^4,$$

$$(2) \quad x = b + y,$$

to find x and y by quadratics. — Communicated by Prof. E. W. Evans, Marietta College, Ohio.

II. Describe an isosceles triangle having each of the angles at the base double of the third angle.

III. Which is greater, $2\tan^{-1}(\sqrt{2}-1)$, or $3\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{5}{99}$? — Communicated by Prof. Richard Cotter, Baltimore, Maryland.

IV. In any spherical triangle

$$\frac{\sin s \sin(s-a)\sin(s-b)\sin(s-c)}{\cos S \cos(S-A)\cos(S-B)\cos(S-C)} = \frac{\cos a \cos b \cos c - 1}{\cos A \cos B \cos C + 1},$$

in which $s = \frac{1}{2}(a+b+c)$, $S = \frac{1}{2}(A+B+C)$.*

V. If the earth be completely covered by a sea of small depth, prove that the depth in latitude l is very nearly equal to

$$H(1 - \varepsilon \sin^2 l),$$

where H is the depth at the equator, and ε the ellipticity of the earth.

Solutions of these problems must be received by May 1st, 1861.

* This problem was misprinted in No. XII., Vol. II.

AWARD OF THE PRIZES FOR SOLUTIONS OF PROBLEMS IN No. II., Vol. III.

The first prize is awarded to George A. Osborne, Jr., Lawrence Scientific School, Cambridge, Mass.

The second prize is awarded to David Trowbridge, Perry City, N. Y.

The third prize is awarded to Asher B. Evans, Madison University, Hamilton, N. Y.

The fourth prize, to junior competitors, is not awarded.

Prize Solution of Problem I.

Prove that the surface of a sphere can be completely covered with the surfaces either of 4, or of 8, or of 20 equilateral spherical triangles.

Suppose either a tetrahedron, an octahedron, or an icosahedron to be inscribed in a sphere. Then planes passing through the edges of the polyhedron and the centre of the sphere will divide its surface into either 4, 8, or 20 equilateral triangles. This solution is given by nearly all the competitors.

The solution of Problem II. by the usual formula is obvious.

Prize Solution of Problem III.

By Asher B. Evans, Madison University, Hamilton, N. Y.

$$\frac{(x+a+b)^5+(x+c+d)^5}{(x+a+c)^5+(x+b+d)^5}=\frac{m}{n}.$$

Find x by quadratics.

Since the sum of the fifth powers of two numbers is divisible by the sum of the numbers, both numerator and denominator of the first member of the given equation is divisible by

$$2x+a+b+c+d.$$

Performing this division, and putting $P=2x+a+b+c+d$ in the result, we get

$$(1)\quad \frac{P^4-5P^2(x+a+b)(x+c+d)+5(x+a+b)^2(x+c+d)^2}{P^4-5P^2(x+a+c)(x+b+d)+5(x+a+c)^2(x+b+d)^2}=\frac{m}{n}.$$

But $x = \frac{P-a-b-c-d}{2}$, which, substituted in (1), gives, after reduction,

$$(2) \qquad \frac{P^4 + 10\,\{(a+b)^2-(c+d)^2\}\,P^2 + 5\,\{(a+b)^2-(c+d)^2\}}{P^4 + 10\,\{(a+c)^2-(b+d)^2\}\,P^2 + 5\,\{(a+c)^2-(b+d)^2\}} = \frac{m}{n}.$$

By putting $p = (a+b)^2-(c+d)^2$, $q = (a+c)^2-(b+d)^2$, (2) becomes

$$(3) \qquad P^4 + \frac{10}{n-m}(np - mq)\,P^2 = \frac{5}{n-m}(mq - np),$$

or

$$(4) \qquad P^4 + 2AP^2 = -A,$$

by putting, $\qquad \frac{5}{n-m}(np - mq) = A.$

From (4) we get the four roots,

$$P = \pm\left(-A \pm \sqrt{(A-\tfrac{1}{2})^2 - \tfrac{1}{4}}\right)^{\frac{1}{2}}.$$

These values of P, substituted in $x = \frac{P-a-b-c-d}{2}$, give four of the values of x, and the fifth is given by putting the divisor $2x + a + b + c + d = 0$.

Therefore $\qquad x = -\frac{a+b+c+d}{2}$

is the fifth root.

W. D. Mershon's solution is similar to the above.

Prize Solution of Problem IV.

By David Trowbridge, Perry City, Schuyler County, N. Y.

A fraudulent tradesman contrives to employ his *false* balance both in buying and selling a certain article, thereby gaining at the rate of 11 per cent more on his outlay than he would gain if the balance were *true*. If however the scale-pans, in which the article is weighed when bought and sold respectively, were interchanged, he would neither gain nor lose by the article. Find the legitimate gain per cent on the article.

Let x denote the fraction of gain or loss in weight by using the false balance, and r the legitimate gain per cent. Every actual unit

of weight weighs $1+x$ when sold, and $1-x$ when bought; he sells each unit for $(1+r)(1+x)$, and buys it for $(1-x)$. His gain is therefore $(1+r)(1+x)-(1-x)$, and the gain per cent is

$$(1) \qquad \frac{(1+r)(1+x)-(1-x)}{1-x} = r + \tfrac{11}{100},$$

by using the false balance. By interchanging the scale-pans he would buy an actual unit of weight for $1+x$, and sell it for $(1+r)(1-x)$; therefore

$$(2) \qquad (1+r)(1-x)-(1+x) = 0,$$

since by the question he neither gains nor loses by the transaction.

Equations (1) and (2) give

$$(3) \qquad (1+r)\frac{(1+x)}{(1-x)} = 1+r+\tfrac{11}{100},$$

$$(4) \qquad \frac{1+x}{1-x} = 1+r.$$

Combining (3) and (4) we get

$$(1+r)^2-(1+r) = \tfrac{11}{100}; \qquad \therefore r = \tfrac{1}{10},$$

or 10 per cent is the legitimate gain.

Prize Solution of Problem V.

By George A. Osborne, Jr., Lawrence Scientific School.

Show that in the parabola the half sum of the radii vectores drawn to the extremities of any arc whatever, is equal to the radius vector drawn to the summit of the diameter passing through the centre of the chord and parallel to the axis, plus the part of this diameter intercepted between the arc and the chord.

Let CD be any arc of the parabola, and F the focus. From C and D, the extremities of the chord CD, and also from its middle point, N, drop perpendiculars, DO, NP, and CQ, upon the directrix AB; and let M be the point in which NP meets the curve. Draw the radii FC, FM, and FD; then by the known property, that any

point in the curve is equally distant from the focus and directrix, we have

$$PN = \tfrac{1}{2}(OD + CQ) = \tfrac{1}{2}(DF + CF)$$
$$= PM + MN = MF + MN;$$
$$\therefore MF + MN = \tfrac{1}{2}(DF + CF).$$

Several of the competitors gave substantially this solution.

Mr. G. B. Hicks gave the following solution: —

Let the abscissas of the extremities of any arc be designated by x_1 and x_2, and the radii vectores drawn to these extremities by r_1 and r_2.

Then calling the equation to the curve $y^2 = 4mx$, we have, from the fundamental property of the parabola,

$$r_1 = x_1 + m, \qquad r_2 = x_2 + m,$$
$$\therefore \tfrac{1}{2}(r_1 + r_2) = \tfrac{1}{2}(x_1 + x_2) + m.$$

Again, let the radius vector drawn to the summit of the diameter passing through the centre of the chord and parallel to the axis be denoted by r_3, and the abscissa of the summit by x_3.

And let h equal the part of the diameter intercepted between the arc and the chord. We have

$$r_3 = x_3 + m.$$

And since the abscissa of the centre of the chord is $\frac{1}{2}(x_1 + x_2)$,

$$h = \tfrac{1}{2}(x_1 + x_2) - x_3;$$
$$\therefore r_3 + h = \tfrac{1}{2}(x_1 + x_2) + m = \tfrac{1}{2}(r_1 + r_2).*$$

---

* One member of the Committee for awarding these Prizes has resigned. We shall hereafter attend to the duty, as it is often inconvenient for the Committee to report in time for the press. Besides, we have no right to demand the continuance of a serious labor so long and so ably performed. — Ed.

FIRST PRIZE ESSAY.—ON THE CONFORMATION OF THE EARTH.

By G. W. Hill, Nyack, N. Y.

1. All the particles which compose the mass of the earth are animated by the attraction of gravitation. The law of this force is, that the attraction of any atom for a spherical surface of material points, described about it as a centre, is constant. Hence, if the attraction of an atom for a material point be represented by A, and r be the radius of the spherical surface, and N the number of material points in a unit of surface, the attraction of the central atom for the spherical surface is $4\pi N r^2 A =$ a constant $= -4\pi N M$. Whence $A = -\frac{M}{r^2}$; that is, the attraction varies inversely as the distance squared. The constant M is called the mass of the attracting atom. We have given A a negative sign because it represents a force tending to decrease the line r.

2. Making $A = \frac{dV}{dr}$, then $V = \frac{M}{r}$. V is called the *potential function*, and has this property: that if the partial differential of it be taken with respect to any of the three spaceal co-ordinates of which it is necessarily a function, the result will be the partial force in the direction of that co-ordinal axis.

3. If the attraction of a single atom give $V = \frac{M}{D}$, denoting the distance by D, the attraction of an indefinite number or assemblage of atoms will give $V = S \cdot \frac{M}{D}$. If ξ, ψ, φ represent any three lines at right angles with each other, then $\frac{dV}{d\xi}, \frac{dV}{d\psi}, \frac{dV}{d\varphi}$ are the forces acting in each of these three directions respectively. In a rectangular system $D = \{(x-x')^2 + (y-y')^2 + (z-z')^2\}^{\frac{1}{2}}$, the accented letters pertain to the attracting atom, the unaccented to the attracted. Consequently,

$$(1)\ \frac{dV}{dx} = S.-\frac{M}{D^3}(x-x'),\ \frac{dV}{dy} = S.-\frac{M}{D^3}(y-y'),\ \frac{dV}{dz} = S.\frac{M}{D^3}(z-z').$$

4. Differentiating again,

$$(2)\qquad \begin{cases} \dfrac{d^2V}{dx^2} = S \cdot \dfrac{M}{D^5}\left\{\dfrac{3}{D^2}(x-x')^2 - 1\right\}, \\ \dfrac{d^2V}{dy^2} = S \cdot \dfrac{M}{D^5}\left\{\dfrac{3}{D^2}(y-y')^2 - 1\right\}, \\ \dfrac{d^2V}{dz^2} = S \cdot \dfrac{M}{D^5}\left\{\dfrac{3}{D^2}(z-z')^2 - 1\right\}. \end{cases}$$

In this differentiation M has been regarded as independent of x, y, z; but in order to render equations (2) altogether general, the attracting mass must be considered as extending into the point x, y, z. Let ρ be the density of the atom occupying this point, which becomes zero when the attracting mass does not reach the point. This atom may be regarded as spherical, then for it $M = \frac{4\pi\rho D^3}{3}$; substituting this value in equations (1), the results are

$$\frac{dV}{dx} = \frac{4\pi\rho}{3}(x'-x), \quad \frac{dV}{dy} = \frac{4\pi\rho}{3}(y'-y), \quad \frac{dV}{dz} = \frac{4\pi\rho}{3}(z'-z).$$

Hence we must add the term $-\frac{4\pi\rho}{3}$ to the right members of equations (2), and then we can regard D as having always a finite value. By adding these equations, there results

$$(3)\qquad \frac{d^2V}{dx^2} + \frac{d^2V}{dy^2} + \frac{d^2V}{dz^2} + 4\pi\rho = 0.$$

The integration of this gives V; ρ is a function of x, y, z; in the case of solid bodies, as the earth, a limited function.

5. Transform (3) to terms of polar co-ordinates; put

$$x = r\sqrt{(1-\mu^2)}\cos\omega, \qquad r = \sqrt{(x^2+y^2+z^2)},$$
$$y = r\sqrt{(1-\mu^2)}\sin\omega, \qquad \mu = \frac{z}{\sqrt{(x^2+y^2+z^2)}},$$
$$z = r\mu, \qquad \omega = \tan^{-1}\frac{y}{x}.$$

Then

$$\frac{d^2V}{dx^2}+\frac{d^2V}{dy^2}+\frac{d^2V}{dz^2}=\left\{\frac{dr^2}{dx^2}+\frac{dr^2}{dy^2}+\frac{dr^2}{dz^2}\right\}\frac{d^2V}{dr^2}+\left\{\frac{d^2r}{dx^2}+\frac{d^2r}{dy^2}+\frac{d^2r}{dz^2}\right\}\frac{dV}{dr}$$
$$+\left\{\frac{d\mu^2}{dx^2}+\frac{d\mu^2}{dy^2}+\frac{d\mu^2}{dz^2}\right\}\frac{d^2V}{d\mu^2}+\left\{\frac{d^2\mu}{dx^2}+\frac{d^2\mu}{dy^2}+\frac{d^2\mu}{dz^2}\right\}\frac{dV}{d\mu}$$
$$+\left\{\frac{d\omega^2}{dx^2}+\frac{d\omega^2}{dy^2}+\frac{d\omega^2}{dz^2}\right\}\frac{d^2V}{d\omega^2}+\left\{\frac{d^2\omega}{dx^2}+\frac{d^2\omega}{dy^2}+\frac{d^2\omega}{dz^2}\right\}\frac{dV}{d\omega},$$
$$=r^{-2}\left\{\frac{d.r^2\frac{dV}{dr}}{dr}+\frac{d.(1-\mu^2)\frac{dV}{d\mu}}{d\mu}+\frac{\frac{d^2V}{d\omega^2}}{1-\mu^2}\right\}.$$

Hence (3) becomes

$$(4)\qquad \frac{d.r^2\frac{dV}{dr}}{dr}+\frac{d.(1-\mu^2)\frac{dV}{d\mu}}{d\mu}+\frac{\frac{d^2V}{d\omega^2}}{1-\mu^2}+4\pi\varrho r^2=0.$$

6. To show the application of (4), take the simple case when the surfaces of equal density are concentrically spherical. Placing the origin of co-ordinates at the common centre, ϱ becomes a function of r alone, either continuous or discontinuous, as the case demands; and evidently $\frac{dV}{d\mu}=0$, $\frac{dV}{d\omega}=0$; therefore (4) becomes

$$\frac{d.r^2\frac{dV}{dr}}{dr}+4\pi\varrho r^2=0.$$

By integration, $\frac{dV}{dr}=(C-4\pi\int\varrho r^2\,dr)r^{-2}$.

$4\pi\int\varrho r^2\,dr$, between the limits 0 and r, is equal to the mass contained within the sphere whose radius is r; denoting this by M, it is clear that $C=0$, since the expression must agree with that for the attraction of a single atom. Thus $\frac{dV}{dr}=-\frac{M}{r^2}$. Or the principle may be stated: "The force acting on any point, wherever situated, equals the mass of all the particles nearer the nucleal centre than the point attracted, divided by the square of the point's distance from that centre, with the negative sign."

7. The earth revolves uniformly about a constant axis; hence, to remove our problem from dynamics to statics, it is necessary to in-

troduce the force of pressure called the centrifugal force. Making the co-ordinal axis of z coincide with the earth's axis, and T denoting the period of its revolution, the potential of the centrifugal force is

$$V = \frac{2\pi^2}{T^2}(x^2+y^2) = \frac{2\pi^2}{T^2} r^2(1-\mu^2).$$

Since this force animates every particle, include its potential in V, and make V the potential of both gravitating and centrifugal force. It then becomes necessary to add to (4) the term $-\frac{8\pi^2}{T^2} r^2$,

$$(5) \quad \frac{d \cdot r^2 \frac{dV}{dr}}{dr} + \frac{d \cdot (1-\mu^2) \frac{dV}{d\mu}}{d\mu} + \frac{\frac{d^2V}{d\omega^2}}{1-\mu^2} + 4\pi\left(\rho - \frac{2\pi}{T^2}\right) r^2 = 0.$$

8. To apply this equation to the solution of the earth's conformation, we must combine it with some condition of equilibrium. From the manner in which the atmosphere and ocean cover the earth, we may conjecture it was once fluid, and in solidifying preserved the form it had taken by the laws of hydrostatics. In passing from the solid earth to the ocean, and from the ocean to the atmosphere, there occur two faults in the continuity of the earth's density; hence ρ is strictly represented by a discontinuous function. But as the mass of the ocean and atmosphere is about $\frac{1}{4000}$ of the whole, its influence may be neglected, and ρ supposed continuous from centre to surface.

9. If p is the pressure, then $dp = \rho\, dV$, and $V + B = 0$, is the equation to surfaces of level, B having a different value for every surface. Let ρ be a function of p, and thus of V. In (5) make

$$4\pi\left(\rho - \frac{2\pi}{T^2}\right) = f(V).$$

10. The centrifugal force being small compared with gravity, may be regarded as a perturbing force. Supposing at first this force is zero, the particles would arrange themselves symmetrically about a centre,

since there is no reason why they should accumulate in one place rather than in another. Take the origin of co-ordinates at this centre, then $\frac{dV}{d\mu} = 0$, $\frac{dV}{d\omega} = 0$. Thus (5) becomes

$$\frac{d \cdot r^2 \frac{dV}{dr}}{dr} + r^2 f(V) = 0.$$

Consequently V is a function of r alone, and the general equation to surfaces of level, $V + B = 0$, when solved gives $r =$ a constant; these surfaces are, then, concentrically spherical.

11. The term $\frac{2\pi^2}{T^2} r^2 (1 - \mu^2)$, which the centrifugal force introduces into V, and which causes a departure from the spherical form, does not contain ω, and so whatever derangement it may produce, cannot introduce ω into V; that is, the earth is a solid of revolution. Consequently (5) becomes

$$\frac{d \cdot r^2 \frac{dV}{dr}}{dr} + \frac{d \cdot (1-\mu^2) \frac{dV}{d\mu}}{d\mu} + r^2 f(V) = 0. \tag{6}$$

12. From the form of this same term it may be concluded that

$$V = Y_0 + Y_1 \mu^2 + Y_2 \mu^4 + \text{\&c.} \ldots = \Sigma \cdot Y_i \mu^{2i}, \tag{7}$$

where Y_i is a function of r alone, and a quantity of the order of the ith power of the centrifugal force. Substitute this expression of V in (6) and put the coefficient of μ^{2i} resulting equal to zero, and let the coefficient of μ^{2i} in $r^2 f(V)$ be U_i, then

$$\frac{d \cdot r^2 \frac{dY_i}{dr}}{dr} + (2i+1)(2i+2) Y_{i+1} - 2i(2i+1) Y_i + U_i = 0. \tag{8}$$

This equation has the inconvenience of introducing Y_{i+1}; let us therefore assume more generally $V = \Sigma \cdot V_i M_i$, V_i a function of r, and M_i of μ, V_i being of the same order as Y_i. Substituting in (6),

$$\Sigma \cdot \frac{d \cdot r^2 \frac{dV_i}{dr}}{dr} M_i + \Sigma \cdot V_i \frac{d \cdot (1-\mu^2) \frac{dM_i}{d\mu}}{d\mu} + r^2 f(\Sigma \cdot V_i M_i) = 0,$$

which may be written

$$(9) \qquad \Sigma . \frac{d . r^2 \frac{dV_i}{dr}}{dr} M_i + \Sigma . V_i \frac{d . (1-\mu^2) \frac{dM_i}{d\mu}}{d\mu} + \Sigma . T_i M_i = 0.$$

In order that the left-hand member may be arranged in a series of the same form as $\Sigma . V_i M_i$, we must have

$$\frac{d . (1-\mu^2) \frac{dM_i}{d\mu}}{d\mu} = n M_i ,$$

in which n is independent of μ. We may determine n from the consideration that V_i being of the same order as Y_i, M_i cannot contain any higher power of μ than μ^{2i}. Making $M_i = \Sigma . k_s \mu^{2s}$, this relation results: $k_{s+1} = \frac{n+2s(2s+1)}{(2s+1)(2s+2)} k_s$. To make this series end at k_i, n must equal $-2i(2i+1)$; and $k_{s+1} = -\frac{(2i-2s)(2i+2s+1)}{(2s+1)(2s+2)} k_i$; hence, putting $k_0 = 1$, which is allowable,

$$(10) \quad M_i = 1 - \frac{2i(2i+1)}{1.2} \mu^2 + \frac{(2i-2)2i(2i+1)(2i+3)}{1.2.3.4} \mu^4 \ldots \pm \frac{2 \ldots 2i(2i+1) \ldots (4i+1)}{1.2.3 \ldots 2i} \mu^{2i}.$$

For V_i we have from (9), by rejecting the sign Σ and dividing by M_i, this equation: —

$$(11) \qquad \frac{d . r^2 \frac{dV_i}{dr}}{dr} - 2i(2i+1) V_i + T_i = 0.$$

From the expression (10) we easily deduce

$$(12) \quad Y_i = \pm \frac{(2i+1)(2i+3) \ldots (4i-1)}{1.3 \ldots (2i-1)} \left\{ V_i + \frac{(i+1)(4i+1)}{1.(2i+1)} V_{i+1} + \frac{(i+1)(i+2)(4i+1)(4i+3)}{1.2.(2i+1)(2i+3)} V_{i+2} + \&c. \ldots \right\}.$$

The reversion of which is

$$(13) \quad V_i = \pm \frac{1.3 \ldots (2i-1)}{(2i+1)(2i+3) \ldots (4i-1)} \left\{ Y_i + \frac{(i+1)(2i+1)}{1.(4i+3)} Y_{i+1} + \frac{(i+1)(i+2)(2i+1)(2i+3)}{1.2.(4i+3)(4i+5)} Y_{i+2} + \&c. \ldots \right\}.$$

The upper sign is to be used when i is even, the lower when odd. From (10) we obtain

$$M_1^2 = \tfrac{4}{5} - \tfrac{4}{7} M_1 + \tfrac{2}{3}\tfrac{7}{5} M_2. \tag{14}$$

From this and the equation $r^2 f(\Sigma . V_i M_i) = \Sigma . T_i M_i$, pursuing the approximation to quantities of the second order, we get these expressions for T_i,

$$\begin{cases} r^{-2} T_0 = f(V_0) + \tfrac{2}{5} V_1^2 f''(V_0) + \text{\&c.} \ldots \\ r^{-2} T_1 = V_1 f'(V_0) - \tfrac{2}{7} V_1^2 f''(V_0) + \text{\&c.} \ldots \\ r^{-2} T_2 = V_2 f'(V_0) + \tfrac{2}{7}\tfrac{7}{0} V_1^2 f''(V_0) + \text{\&c.} \ldots \end{cases} \tag{15}$$

If quantities of the second order are neglected, the two differential equations to be integrated are

$$\begin{cases} \frac{d . r^2 \frac{d V_0}{dr}}{dr} + r^2 f(V_0) = 0, \\ \frac{d . r^2 \frac{d V_1}{dr}}{dr} - 6 V_1 + r^2 V_1 f'(V_0) = 0. \end{cases} \tag{16}$$

To pursue the analysis farther would require a knowledge of the form of $f(V)$.

13. However, when the point r, μ, ω is without the surface of the earth, (11) can be integrated. Supposing the point not to partake of the motion of revolution, the centrifugal force must be neglected, and since $\rho = 0$, generally $T_i = 0$; consequently (11) becomes

$$\frac{d . r^2 \frac{d \breve{V}_i}{dr}}{dr} - 2i(2i+1) \breve{V}_i = 0. \tag{17}$$

The integral of this is $\breve{V}_i = a_i r^{2i} + b_i r^{-2i+1}$, in which $a_i = 0$; otherwise the earth's attraction would be infinitely great at an infinite distance. From the equation $V = \Sigma . V_i M_i$, by giving i successively the values 0, 1, 2, &c., we have, then,

$$\breve{V} = b_0 r^{-1} + b_1 r^{-3}(1 - 3\mu^2) + b_2 r^{-5}(1 - 10\mu^2 + \tfrac{3}{3}\tfrac{5}{5}\mu^4) + \text{\&c.} \ldots \tag{18}$$

We have written V in this case $\breve{V}$. If r, μ, ω is situated in the atmosphere or at the surface, and revolves with the earth, we have only to add to $\breve{V}$ the potential of the centrifugal force $\frac{2\pi^2}{T^2} r^2 (1-\mu^2)$, and may then call it $\bar{V}$; and for points within write $\hat{V}$.

14. The integration of (11) introduces an indefinite number of constants, of which the superfluous are to be eliminated in the usual way of treating partial differential equations, viz.: by equations of limitation; which in the present question are the equations having places at the limit of the attracting mass. At this limit $\frac{dV}{dr}$, $\frac{dV}{d\mu}$, $\frac{dV}{d\omega}$, derived from the integration of (5), must have the same value, whether ϱ in that equation equals zero or its value at the surface. Consequently we have

$$(19) \qquad \frac{d.(\hat{V}-\bar{V})}{dr} = 0, \qquad \frac{d.(\hat{V}-\bar{V})}{d\mu} = 0, \qquad \frac{d.(\hat{V}-\bar{V})}{d\omega} = 0.$$

Hydrostatics furnishes two other equations,

$$(20) \qquad \hat{V} + B = 0, \qquad \bar{V} + B' = 0.$$

These five are all equations to the same surface, of which it must be noticed that they are not all independent, but any one is a consequence of the other four. They will give all the conditions needed to eliminate the superfluous arbitrary constants. The last of (19) is, since V does not involve ω, an identical equation; select then the first of (19) and the two of (20) for the purpose of elimination. It is evident from $V = \Sigma . V_i M_i$, that each one of these equations to the surface can be transformed to this shape: $r = \Sigma . c_i M_i$, c_i a constant and of the ith order with respect to the centrifugal force. By substituting this value of r in equations (19) and (20), and equating the coefficients of M_i generally to zero, and rejecting the two involving B and B', we have, pursuing the approximation to the ith order, $3i+1$ equations. The constants they involve are the con-

stants resulting from the integration of (11), $2i+2$ in number, and the constants c_i, $i+1$ in number, in all $3i+3$. Thus two constants are left indeterminate, which is as it should be, since the units of length and density are arbitrary. We may assume, then, $c_0 = 1$; making this substitution in $\widehat{V} + B = 0$, using accents to denote differentiation and putting r afterwards equal to unity, and taking account of quantities of the second order, we have

$$(21) \qquad \begin{cases} V_0' c_1 + V_1 - \frac{2}{7} c_1 \{ V_0'' c_1 + 2 V_1' \} = 0. \\ V_0' c_2 + V_2 + \frac{2}{7}\frac{7}{0} c_1 \{ V_0'' c_1 + 2 V_1' \} = 0. \end{cases}$$

Treating $\overline{V} + B' = 0$ in the same way, two similar equations result, which can be evidently derived from (21) by making

$$V_0 = b_0 r^{-1} + \frac{4\pi^2}{3T^2} r^2, \quad V_1 = b_1 r^{-3} + \frac{2\pi^2}{3T^2} r^2, \quad \text{and} \quad V_2 = b_2 r^{-5},$$

$$(22) \qquad \begin{cases} \left\{ -b_0 + \frac{8\pi^2}{3T^2} \right\} c_1 + b_1 + \frac{2\pi^2}{3T^2} - \frac{2}{7} c_1 \left\{ 2 b_0 c_1 - 6 b_1 + \frac{8\pi^2}{3T^2} \right\} = 0, \\ -b_0 c_2 + b_2 + \frac{2}{7}\frac{7}{0} c_1 \left\{ 2 b_0 c_1 - 6 b_1 + \frac{8\pi^2}{3T^2} \right\} = 0. \end{cases}$$

From the equation $\frac{d.(\widehat{V} - \overline{V})}{dr} = 0$, also we have

$$(23) \qquad \begin{cases} V_0' + \frac{2}{5} c_1 \{ V_0''' c_1 + 2 V_1'' \} = -b_0 + \frac{8\pi^2}{3T^2} + \frac{2}{5} c_1 \left\{ -6 b_0 c_1 + 24 b_1 + \frac{8\pi^2}{3T^2} \right\}, \\ V_0'' c_1 + V_1' - \frac{2}{7} c_1 \{ V_0''' c_1 + 2 V_1'' \} = \left(2 b_0 + \frac{8\pi^2}{3T^2} \right) c_1 - 3 b_1 + \frac{4\pi^2}{3T^2} \\ \qquad\qquad - \frac{2}{7} c_1 \left\{ -6 b_0 c_1 + 24 b_1 + \frac{8\pi^2}{3T^2} \right\}, \\ V_0'' c_2 + V_2' + \frac{2}{7}\frac{7}{0} c_1 \{ V_0''' c_1 + 2 V_1'' \} = 2 b_0 c_2 - 5 b_2 \\ \qquad\qquad + \frac{2}{7}\frac{7}{0} c_1 \left\{ -6 b_0 c_1 + 24 b_1 + \frac{8\pi^2}{3T^2} \right\}. \end{cases}$$

Equations (22) and (23) can be reduced to the following simpler forms: —

$$(24)\quad \begin{cases} b_0 = -\{1 + \frac{2}{3}q + \frac{1}{3}q^2 - \frac{4}{3}q\,c_1 + \frac{26}{5}c_1^2\}\ \{V_0' + \frac{2}{5}c_1\,[V_0'''\,c_1 + 2\,V_1'']\}, \\ b_1 = \left\{(1 - \frac{4}{21}q)\,c_1 - \frac{2}{7}c_1^2 - \frac{q}{6}\left(1 - \frac{q}{2}\right)\right. b_0, \\ b_2 = \{c_2 - \frac{9}{14}q\,c_1 + \frac{34}{35}c_1^2\}\,b_0, \end{cases}$$

$$(25)\quad \begin{cases} V_0''\,c_1 + V_1' - \frac{2}{7}c_1\{V_0'''\,c_1 + 2V_1''\} - \{(1 - \frac{32}{21}q)\,c_1 + \frac{13}{7}c_1^2 - \frac{5}{6}q\left((1 + \frac{q}{6})\right\}V_0' = 0 \\ V_0''\,c_2 + V_2' + \frac{27}{70}c_1\{V_0'''\,c_1 + 2\,V_1''\} - \{3c_2 - \frac{27}{14}q\,c_1 + \frac{27}{35}c_1^2\}\,V_0' = 0, \end{cases}$$

if we make $\frac{4\pi^2}{T^2} = q\,\{b_0(1 - 3\,c_1) + 3\,b_1\}$.

15. In this article we shall neglect quantities of the order of q^2. Let $\frac{w}{3}$ denote the quantity obtained by dividing the second member of the second equation of (23) by the second member of the first equation, then

$$w = 3\,.\,\frac{-3\,b_1 + \frac{4\pi^2}{3T^2} + 2\,b_0 c_1}{b_0} = 6\,c_1 + q - 9\,\frac{b_1}{b_0};$$

from the second equation of (24), $3\,c_1 = \frac{3\,b_1}{b_0} + \frac{q}{2} = e$, the approximate value of the earth's compression, as is clear from the equation $r = 1 + c_1(1 - 3\,\mu^2) + \&c....$ Hence by addition

$$(26)\qquad e + w = 6\,c_1 + \tfrac{3}{2}q - 6\,\frac{b_1}{b_0} = \tfrac{5}{2}q.$$

The relation enunciated by this equation is known as CLAIRAUT'S Theorem.

16. The equation of the earth's surface is

$$\begin{aligned} r &= 1 + c_1(1 - 3\,\mu^2) + c_2(1 - 10\,\mu^2 + \tfrac{35}{3}\mu^4) + \&c.... \\ &= (1 + c_1 + c_2)\{1 - [3\,c_1(1 - c_1) + 10\,c_2]\,\mu^2 + \tfrac{35}{3}c_2\mu^4 + \&c....\} \\ &= a\,\{1 + e_1 \sin^2\theta + e_2 \sin^4\theta + \&c. ...\}\,; \end{aligned} \qquad (27)$$

in which θ is the geocentric latitude. If e represent the compression, or part of its own length by which the equatorial radius exceeds the polar,

$$(28)\qquad e = -e_1 - e_2 = 3\,c_1(1 - c_1) - \tfrac{5}{3}c_2.$$

17. Denoting the normal or astronomical latitude by θ', we have

$$(29)\quad \theta' = \theta - \tan^{-1}\frac{dr}{r\,d\theta} = \theta - \left\{e_1 + e_2 - \frac{e_1^2}{2}\right\}\sin 2\theta - \frac{2e_2 - e_1^2}{4}\sin 4\theta,$$

the reversion of which is

$$(30)\quad \theta = \theta' + \left\{e_1 + e_2 - \frac{e_1^2}{2}\right\}\sin 2\theta' + \frac{2e_2 + 3e_1^2}{4}\sin 4\theta'.$$

18. A line geodetically measured on the earth's surface is clearly the shortest possible; hence, if ds is the element of the curve, by the principles of the Calculus of Variations, $d.\int ds + \lambda\,\delta V = 0$, λ being the indeterminate multiplier of δV. Also,

$$ds^2 = dr^2 + \frac{r^2\,d\mu^2}{1-\mu^2} + r^2(1-\mu^2)\,d\omega^2.$$

The coefficients of δr, $\delta\mu$, $\delta\omega$ each equal zero; retaining only that of $\delta\omega$, as sufficient for our purpose, we have

$$d.\left\{r^2(1-\mu^2)\frac{d\omega}{ds}\right\} = 0, \quad \text{or} \quad r^2(1-\mu^2)\frac{d\omega}{ds} = ah.$$

That is, the sine of the angle made by this curve with the meridians varies inversely as the distance from the earth's axis. Hence

$$\frac{d\omega}{d\mu} = \frac{ah\sqrt{1+\left(\frac{dr}{r\,d\mu}\right)^2(1-\mu^2)}}{(1-\mu^2)\sqrt{r^2(1-\mu^2)-a^2h^2}} \quad \text{and} \quad \frac{ds}{d\mu} = \frac{r^2\sqrt{1+\left(\frac{dr}{r\,d\mu}\right)^2(1-\mu^2)}}{\sqrt{r^2(1-\mu^2)-a^2h^2}}.$$

Taking account only of the first order with respect to q,

$$d\omega = \frac{h\,d\mu}{(1-\mu^2)\sqrt{(1-h^2-\mu^2)}} - \frac{h\,e_1\mu^2\,d\mu}{(1-h^2-\mu^2)^{\frac{3}{2}}}.$$

$$(31)\quad \omega = C - \sin^{-1}\frac{1-\frac{1+h^2}{1-h^2}\mu^2}{1-\mu^2} + h\,e_1\left\{\sin^{-1}\frac{\mu}{\sqrt{(1-h^2)}} - \frac{\mu}{\sqrt{(1-h^2-\mu^2)}}\right\}.$$

This, then, is the equation to the curve; if $\omega_{,}$, $\omega_{,,}$ are the extreme values of ω, and $\mu_{,}$, $\mu_{,,}$ those of μ, h can be found from the expression

for $\omega_{,,} - \omega_{,}$. If ε is the angle made by the curve with the meridian at the commencement, then $h = \cos\theta_{,} \sin\varepsilon\,(1 + e_1 \sin^2\theta_{,})$, and as an approximate value of ε, we have

$$\cot\varepsilon = \frac{\tan\theta_{,,}\cos\theta_{,}}{\sin(\omega_{,,}-\omega_{,})} - \sin\theta_{,}\cot(\omega_{,,}-\omega_{,}).$$

For the length of the curve,

$$ds = a\left\{\frac{1+e_1\mu^2}{\sqrt{(1-h^2-\mu^2)}} - \frac{h^2 e_1 \mu^2}{(1-h^2-\mu^2)^{\frac{3}{2}}}\right\} d\mu,$$

$$(32)\quad s = a\left\{\left[1+(1+h^2)\frac{e_1}{2}\right]\sin^{-1}\frac{\mu}{\sqrt{(1-h^2)}} - \frac{e_1}{2}\frac{(1+h^2-\mu^2)\mu}{\sqrt{(1-h^2-\mu^2)}}\right\}.$$

If $h = 0$, this expression gives the length of any arc of a meridian, but in this case,

$$ds^2 = \sqrt{(dr^2 + r^2 d\theta^2)} = r\left\{1 + \frac{e_1^2}{2}\sin^2 2\theta\right\} d\theta,$$

or

$$ds = a\left\{1 + e_1\sin^2\theta + e_2\sin^4\theta + \frac{e_1^2}{2}\sin^2 2\theta\right\} d\theta,$$

$$(33)\quad s = a\left\{\left(1+\frac{e_1}{2}+\frac{e_1^2}{4}+\frac{3}{8}e_2\right)\theta - \frac{e_1+e_2}{4}\sin 2\theta + \frac{2e_1^2-e_2}{32}\sin 4\theta\right\}.$$

19. All areas on the earth's surface bounded by lines, whose equation is (31), can be divided into a finite number of parts, each contained by an arc of a meridian, an arc of a parallel of latitude, and a line whose equation is (31). Let A denote the area of this, then ds, being the element of the meridian, $A = \int \omega r \cos\theta\, ds$, or, neglecting quantities of the second order, $= \int \omega r^2 d\mu = ar\mu\omega - a\int r\mu\, d\omega$, finally putting for $d\omega$ its value, and integrating: —

$$(34)\quad A = ar\mu\omega + \frac{a^2 h}{2}\left\{\frac{1+e_1}{h}\tan^{-1}\frac{\sqrt{(1-h^2-\mu^2)}}{h} + \frac{(1-h^2)e_1}{\sqrt{(1-h^2-\mu^2)}}\right\}.$$

20. Let F denote the force of gravity at the surface, then

$$F = \frac{dV}{dr}\sqrt{1+\left(\frac{dr}{rd\theta}\right)^2} = \frac{dV}{dr}\{1 + 18\,e_1^2\mu^2(1-\mu^2)\}.$$

If A_0, A_1, A_2 are the numerical values of the members of equations (23), then

$$F = (A_0 + A_1 + A_2)\left\{1 - \frac{3A_1 + 10A_2 - 18V_0'c_1^2}{A_0 + A_1}\mu^2 + \frac{\frac{35}{3}A_2 - 18V_0'c_1^2}{A_0}\mu^4\right\}.$$

$$(35)\quad = F_0\{1 + w_1\mu^2 + w_2\mu^4 + \text{\&c.}\ldots\}.$$

21. Thus far the general theory of the subject. We shall now assume some particular law of density. Suppose that the matter of which the earth is composed is compressible inversely as its density. This gives $d\varrho = \frac{m^2}{4\pi}\frac{dp}{\varrho}$, m being a constant. Substituting for dp its value, $\varrho\, dV$, and integrating $\varrho = \frac{m^2}{4\pi}V$, no constant is added, because it may be supposed contained in V. Then

$$f(V) = m^2 V - \frac{8\pi^2}{T^2},$$

and from (15) generally $T_i = m^2 r^2 V_i$, but

$$T_0 = m^2 r^2\left(V_0 - \frac{8\pi^2}{m^2 T^2}\right).$$

Thus (11) becomes

$$(36)\qquad \frac{d.r^2\frac{dV_i}{dr}}{dr} - \{2i(2i+1) - m^2r^2\}V_i = 0.$$

In integrating, the part of V_i involving negative powers of r may be neglected, since it belongs to $\breve{V}$. If a_i is an arbitrary constant, and f^{2i} denotes the operation $\frac{d.}{dr}\frac{1}{r}$ performed $2i$ times, the integral of (36) is $V_i = a_i r^{2i-1} f^{2i}(\sin mr)$. The integral may also be obtained thus: put (36) under this form, —

$$(37)\qquad \frac{d^2.rV_i}{dr^2} + \left\{m^2 - \frac{2i(2i+1)}{r^2}\right\}rV_i = 0.$$

Assume $rV_i = P\sin mr + P'\cos mr$; which by substitution gives

$$\frac{d^2.(P \pm P')}{dr^2} \pm 2m\frac{d.(P \mp P')}{dr} - \frac{2i(2i+1)}{r^2}(P \pm P') = 0.$$

Make $P \pm P' = \beta_0 \pm \beta_1 r^{-1} + \beta_2 r^{-2} \pm \beta_3 r^{-3} +$ &c. ...; then this equation results,

$$\pm 2m(n+1)\beta_{n+1} + \{n(n+1) - 2i(2i+1)\}\beta_n = 0;$$

whence $$\beta_{n+1} = \mp \frac{(n-2i)(n+2i+1)}{(n+1)\,2m}\beta_n,$$

the upper sign being taken when n is even, the lower when odd. Then making $\beta_0 = \pm m^{2i} a_i$, in order to agree with the expression $V_i = a_i r^{2i-1} f^{2i}(\sin mr)$, we have

$$(38) \qquad \beta_n = \pm \frac{(2i-n+1)\ldots(2i+n)}{1.2\ldots n.2^n} m^{2i-n} a_i;$$

the upper sign having place when $2i-n$ is of the forms $4\nu+2$, $4\nu+3$, the lower when it is of the forms 4ν, $4\nu+1$.

$$(39) \quad V = \frac{8\pi^2}{m^2 T^2} + a_0 r^{-1} f^0(\sin mr) + a_1 r f^2(\sin mr)\{1 - 3\mu^2\} + a_2 r^3 f^4(\sin mr)\{1 - 10\mu^2 + \tfrac{35}{3}\mu^4\} + \text{&c.}\ldots;$$

or, expanding $f^{2i}(\sin mr)$ by using (38),

$$(40) \quad V = \frac{8\pi^2}{m^2 T^2} + a_0 \frac{\sin mr}{r} + a_1\left\{\left(\frac{3}{r^2} - m^2\right)\frac{\sin mr}{r} - \frac{3m}{r}\frac{\cos mr}{r}\right\}\{1 - 3\mu^2\} + a_2\left\{\left(\frac{105}{r^4} - \frac{45m^2}{r^2} + m^4\right)\frac{\sin mr}{r} - \left(\frac{105m}{r^3} - \frac{10m^3}{r}\right)\frac{\cos mr}{r}\right\}\{1 - 10\mu^2 + \tfrac{35}{3}\mu^4\} + \text{&c.}\ldots$$

22. Since V contains a constant, the term $\frac{8\pi^2}{m^2 T^2}$ may be neglected, except in finding the value of the density. Moreover, for simplicity, let $a_0 = 1$, and $\frac{V_0'}{m^2 V_0} = -H$; then from (36) and (40) we derive

$$(41) \quad \begin{aligned} V_0'' &= (2H - 1)m^2 V_0, \\ V_0''' &= (2 + (m^2 - 6)H)m^2 V_0, \\ V_1 &= a_1(3H - 1)m^2 V_0, \end{aligned}$$

$$V_1' = a_1 (3 + (m^2 - 9) H) m^2 V_0,$$
$$V_1'' = a_1 (m^2 - 12 - (5 m^2 - 36) H) m^2 V_0,$$
$$V_2 = a_2 (m^2 - 35 - (10 m^2 - 105) H) m^2 V_0,$$
$$V_2' = a_2 (-10 m^2 + 175 - (m^4 - 65 m^2 + 525) H) m^2 V_0.$$

By substituting these expressions in (21) and (25), neglecting quantities of the second order, they become, after removing the factor $m^2 V_0$,

$$(3H - 1) a_1 - H c_1 = 0,$$

and

$$\{3 + (m^2 - 9) H\} a_1 + (3H - 1) c_1 - \tfrac{5}{6} q H = 0;$$

whence

$$(42) \quad a_1 = \tfrac{5}{6} q \frac{H^2}{m^2 H^2 - (3H - 1)}, \qquad c_1 = \tfrac{5}{6} q \frac{H(3H - 1)}{m^2 H^2 - (3H - 1)}.$$

23. Represent the volume of the earth by v, its superficial density by R, its mean density by R'. Then true to quantities of the first order, $v = \frac{4\pi}{3}$; and if (V) denote the value of V at the surface $R = \frac{m^2}{4\pi}(V)$. Since b_0 is the mass of the earth, $R' = \frac{b_0}{v} = \frac{3 b_0}{4\pi}$. Hence $\frac{m^2 (V)}{b_0} = \frac{3R}{R'}$, or, putting for b_0 its value from (24),

$$\frac{V_0'}{m^2 (V)} = - \frac{R'}{3R (1 + \frac{2}{3} q)}.$$

If in the expressions for V we make $3\mu^2 = 1$, and consequently $r = 1$,

$$(V) = \frac{8\pi^2}{m^2 T^2} + V_0 = - \frac{2q}{m^2} V_0', \quad \text{then} \quad \frac{H}{1 + 2qH} = \frac{R'}{3R(1 + \frac{2}{3} q)}, \quad \text{or}$$

$$(43) \qquad H = \frac{R'}{3R - 2q(R' - R)}.$$

To find m we have, since $H = - \frac{V_0'}{m^2 V_0}$,

$$(44) \qquad \frac{1}{m} \left(\frac{1}{m} - \cot m \right) = H.$$

24. In order to test the preceding theory by numerical calculation, we adopt the following values for q, R, R', the best we can find: —

$$q = \tfrac{1}{288}, \qquad R = 2.55, \qquad R' = 5.67.$$

We shall mark with an accent the numbers of the formulas from which the numerical values of the following quantities are obtained: —

(43)′ $H = 0.739083,$

(44)′ $\begin{cases} m &= 146^\circ\ 12'\ 19''.8, \\ &= 2.551765, \end{cases}$

(42)′ $\begin{cases} a_1 &= 0.00067556, \\ c_1 &= 0.0011126, \end{cases}$

(41)′ $V_0'' = 0.478166\, m^2 V_0,$

(41)′ $\begin{cases} V_0''' = 2.378\, m^2 V_0, \\ V_1 = 1.217249\, m^2 V_0 a_1, \\ V_1' = 1.160794\, m^2 V_0 a_1, \\ V_1'' = -2.944\, m^2 V_0 a_1, \\ V_2 = 0.990\, m^2 V_0 a_2, \\ V_2' = 3.345\, m^2 V_0 a_2. \end{cases}$

To obtain the values of a_1, c_1 true to quantities of the order of q^2, by substituting the preceding in (21) and (25), we have

(21)′ $0.739083\, c_1 - 1.217249\, a_1 + 0.000000666 = 0,$

(25)′ $1.217249\, c_1 + 1.160794\, a_1 - 0.002141513 = 0.$

(21)′ $0.739\, c_2 - 0.990\, a_2 - 0.000000900 = 0,$

(25)′ $2.695\, c_2 + 3.345\, a_2 - 0.000005243 = 0.$

$$a_1 = 0.000676848, \qquad c_1 = 0.001113848,$$
$$a_2 = 0.000000283, \qquad c_2 = 0.000001595.$$

(18)′ & (24)′ $\breve{V} = b_0 \{ r^{-1} + 0.0005430\, r^{-3}(1 - 3\mu^2) + 0.0000010\, r^{-5}(1 - 10\mu^2 + \tfrac{35}{3}\mu^4) \}.$

(27)′ $r = a\{1 - 0.003353781 \sin^2\theta + 0.000018608 \sin^4\theta\}.$

(28)′ $e = 0.003335173 = \dfrac{1}{299.834}.$

(29)′ $\theta' = \theta + 679''.09 \sin 2\theta - 1''.34 \sin^4\theta.$

(30)′ $\theta = \theta' - 679''.09 \sin 2\theta' + 3''.66 \sin^4\theta'.$

(33)′ $s = a\{0.9983329\,\theta + 0.0008338 \sin 2\theta + 0.0000001 \sin 4\theta\}.$

(35)′ $F = F_0\{1 + 0.0054093 \sin^2\theta - 0.0000539 \sin^4\theta\}.$

The following table contains the values of ϱ and of e, the compression of the surfaces of level, for every tenth of the equatorial radius, calculated from the equations

$$\varrho = \frac{m^2}{4\pi} V = \frac{R \sin mr}{r \sin m}, \quad \text{and} \quad e = -\frac{3 V_1}{V_0'} = 3 a_1 \left\{ \frac{3}{r} - \frac{m}{\frac{1}{mr} - \cot mr} \right\}.$$

$\frac{r}{a}$	ϱ	e	$\frac{r}{a}$	ϱ	e
0.0	11.699	0000	0.6	7.635	1–586th
0.1	11.572	1–3675th	0.7	6.398	1–488th
0.2	11.197	1–1872th	0.8	5.108	1–412th
0.3	10.589	1–1240th	0.9	3.810	1–349th
0.4	9.770	1–915th	1.0	2.550	1–300th
0.5	8.773	1–720th			

NOTES AND QUERIES.

I. A Logical Outline of Arithmetic. — One great error in our systems of elementary instruction is the lack of logical method. With the desire of lending a word toward the correction of this error, we present the following logical outline of Arithmetic.

All numerical ideas begin with the unit. It is the origin, the basis of Arithmetic. The unit can be increased and divided, hence arise Numbers and Fractions. Every number is a *synthesis*, every fraction the result of an *analysis*. When by synthesis we have obtained a number, by analysis we may pass to a smaller number; hence numbers can be increased and diminished, and these are the only operations to which they may be subjected. All other processes are merely modifications of, or are based upon, these fundamental ones.

To determine *when* and *how* to increase, and *when* and *how* to diminish, we employ a process of reasoning called Comparison. This

reasoning process also gives rise to several particular arithmetical processes. Arithmetic of whole numbers — and the same obtains with fractions — is therefore reduced to the two general processes of Synthesis and Analysis, these to be controlled by the logic of Comparison. Let us glance briefly at each.

A general synthesis is Addition. Multiplication is a special case of addition, in which the numbers to be added are equal, — the sum in this case being distinguished by the term *product.* Composite numbers, formed by the synthesis of factors; Multiples formed by the synthesis of *particular* factors, and Involution, requiring a product of *equal* factors, are all included under Multiplication.

A general analysis is called Subtraction. Division is a *special* analysis, in which equal numbers are successively subtracted, with the additional object of ascertaining how many times such subtractions may be performed. Factoring is a special case of division, in which many or all of the divisors of a member are required; Evolution, a special case of factoring, in which *one* of several *equal* factors is required; and Common Divisor, a case of factoring in which some common factor of several numbers is required.

In comparing numbers we observe we may pass from one to another of different species under the same genus, and thus we have Reduction. By Comparison we obtain Ratio, arithmetical and geometrical, from the first of which, by a farther comparison, arises Arithmetical Progression; and from the second we obtain, in a similar manner, Proportion and Geometrical Progression.

Thus we derive a complete outline of the science of numbers. The rest of Arithmetic consists of the solution of problems, either real or theoretic, and may be included under the head of Applications of Numbers. Arithmetic is therefore pure and applied, or abstract and concrete. The following is our logical outline of abstract Arithmetic: —

Synthesis.	Addition.	{Multiplication.	Forming Composite Numbers.
			Involution.
			Multiples.
Analysis.	Subtraction.	{Division.	Factoring.
			Evolution.
			Common Divisor.
Comparison.	Reduction.		
	Ratio.		
	Proportion.		
	Progressions.		

It is believed that the above view of Arithmetic must tend to simplify the subject, and that much clearer notions of the science will be obtained when these philosophical relations are understood. — Edward Brooks, Prof. of Math., State Normal School, Millersville, Pa.

II. *Note on finding the expressions for the sines and cosines of the sum and difference of two angles in terms of the sines and cosines of the angles themselves, by using the equations for passing from a system of rectangular co-ordinates to another system also rectangular.*

These equations are as follows: —

(1) $$y = x' \sin a + y' \cos a,$$

(2) $$x = x' \cos a - y' \sin a.$$

But $y = \sin(\alpha + \beta)$, and $x = \cos(\alpha + \beta)$.

Also $y' = \sin \beta$, and $x' = \cos \beta$.

Substituting these values in (1) and (2), we have

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha,$$

and $$\cos(\alpha + \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta.$$

If the point P falls between the axes of x, then y' is negative, and the equations become

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha,$$

and $$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$$

— M. C. STEVENS, Professor of Mathematics, Haverford College, Pa.

III. The following similarity in the form of equations may not be uninteresting to some readers of the Monthly.

$\frac{x}{a} + \frac{y}{b} = 1$ is the equation of the straight line,

$\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$ is the equation of ellipse or hyperbola,

according as we take the upper or lower sign, the origin being the centre.

$\frac{xx'}{a^2} \pm \frac{yy'}{b^2} = 1$ is the equation to the tangent line to the ellipse or hyperbola.

In these equations, a is the intercept on the axis of x, and b is the intercept on the axis of y.

In the case of the hyperbola, the intercept on the axis of y is imaginary.

These forms, by their similarity, are easily remembered. I take them from SALMON'S Conic Sections. — M. C. S.

IV. *Notes on Geometry.* — We think that nothing ought to be admitted in Geometry as self-evident which admits of proof. The only basis of geometrical reasoning ought to be the definitions, and those intuitive truths which can be reduced to no simpler elements, and which it is impossible to deny.

Now it can be *proved* that all right angles are equal, and LEGENDRE has given the proof satisfactorily; why, then, is this truth taken for granted in most of our Geometries? It does not follow that because it is sufficiently obvious to some minds, it is so to all.

Again, in order that any assumed measure of the inclination of two planes may be a just measure, it is necessary that the measure shall be equal for equal inclinations, that it shall vary as the inclinations vary, and that it shall be the same at whatever point of the intersection of the two planes it may be applied; and yet, with all these conditions to be satisfied, it is taken for granted that the angle contained between two perpendiculars, one in each plane and perpendicular to the common intersection at the same point, is a just measure of the inclination of the planes. It is, to say the least, quite as self-evident that an angle is measured by the arc intercepted between its sides and described from its vertex as a centre. Then why demonstrate this last?

It is more excusable to assume, that only one line can be drawn through a given line parallel to a given line, because it was very difficult to prove this proposition in an elementary way, and very difficult to establish the theory of parallel lines without it.

The following method seems elementary enough and entirely satisfactory. Let ab and de be two parallels, and ae a line making any angle with ab. Now however small the angle bac, it may be made, by being repeated a definite number of times around a, to fill up the whole plane space about a; it is, therefore, when the sides ac and ab are indefinitely produced, a finite part of that space. But the space between the parallels ab and de, extended indefinitely, can never, by any number of repetitions, be made to fill up the whole plane space, and it must therefore be an infinitely small part of that space. Hence, if ab and ac be sufficiently extended, the space between ab and ac must ultimately become greater than the space between ab and de, which requires that ac shall intersect de, and pass beyond it.

This method, we believe, was first suggested by one of the Bernouillis, and we see no objection to it, except that the idea of infinity

is involved in the reasoning; and yet this same idea meets us in the very definition of parallel lines in the theory of the circle, and in incommensurable quantities.

The mention of incommensurables brings us to our last remark: it is, that LEGENDRE's method of treating these quantities leaves the learner with very vague and indistinct ideas of them. We have two angles, A and B, and their arcs C and D, and we prove the relation $\frac{A}{B} = \frac{C}{D}$. If the learner asks what $\frac{A}{B}$ and $\frac{C}{D}$ mean, we tell him that all ratios are essentially and necessarily *numbers*, and cannot be anything else; but if A and B are incommensurable, $\frac{A}{B}$ cannot be expressed by any conceivable number, nor can $\frac{C}{D}$; we may say that whatever expresses $\frac{A}{B}$ will also express $\frac{C}{D}$; but he may reply that no number can be made to express either, and how shall we get out of the difficulty? Simply by showing from the outset that incommensurable quantities approximate to commensurable as their divisor is made smaller and smaller, and that as the relation $\frac{A}{B} = \frac{C}{D}$ subsists at each step in the approximation, it will continue to be true, however far the process be carried; and yet as $\frac{A}{B}$ and $\frac{C}{D}$ are at each step ratios between commensurable quantities, they are always actual numbers.

To illustrate: suppose $A = 3$ and $B = 5 + R$, then $C = 3$ and $D = 5 + r$; neglecting R and r $\frac{A}{B} = \frac{3}{5} = \frac{C}{D}$. Now take a divisor of A and B smaller than R; C and D will be divided into parts smaller than r; and let

$$A = 7 \text{ and } B = 12 + R', \qquad C = 7 \text{ and } D = 12 + r'.$$

Then neglecting R' and r' $\frac{A}{B} = \frac{7}{12} = \frac{C}{D}$. And so on. — PIKE POWERS, Staunton, Va.

V. *Problem.* — In any triangle, plane or spherical, if α, β, γ denote the perpendiculars dropped from the angles upon the opposite sides,

ϱ, r the radii of the inscribed and circumscribed circles, δ the distance between the centres; $\varrho', \varrho'', \varrho'''$ the radii of the escribed circles, $\delta', \delta'', \delta'''$ the distances between the centres of these circles and the centre of the circle whose radius is r, prove that in the plane

(1) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{1}{\varrho}$, (2) $-\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{1}{\varrho'}$,

(3) $\frac{1}{\alpha}-\frac{1}{\beta}+\frac{1}{\gamma}=\frac{1}{\varrho''}$, (4) $\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}=\frac{1}{\varrho'''}$.

Squaring these four equations, and adding, we get

(5) $$\frac{4}{\alpha^2}+\frac{4}{\beta^2}+\frac{4}{\gamma^2}=\frac{1}{\varrho^2}+\frac{1}{\varrho'^2}+\frac{1}{\varrho''^2}+\frac{1}{\varrho'''^2},$$

Also, as is easily seen,

(6) $$\frac{8}{\alpha\beta}=\frac{1}{\varrho^2}+\frac{1}{\varrho'''^2}-\frac{1}{\varrho'^2}-\frac{1}{\varrho''^2}, \quad \frac{8}{\alpha\gamma}=\frac{1}{\varrho^2}+\frac{1}{\varrho''^2}-\frac{1}{\varrho'^2}-\frac{1}{\varrho'''^2},$$
$$\frac{8}{\beta\gamma}=\frac{1}{\varrho^2}+\frac{1}{\varrho'^2}-\frac{1}{\varrho''^2}-\frac{1}{\varrho'''^2}.$$

In the sphere,

(1) $$\frac{1}{\sin\alpha}+\frac{1}{\sin\beta}+\frac{1}{\sin\gamma}=\frac{1}{\sin\varrho}\,\frac{\cos\delta}{\cos r},$$

(2) $$-\frac{1}{\sin\alpha}+\frac{1}{\sin\beta}+\frac{1}{\sin\gamma}=\frac{1}{\sin\varrho'}\,\frac{\cos\delta'}{\cos r},$$

(3) $$\frac{1}{\sin\alpha}-\frac{1}{\sin\beta}+\frac{1}{\sin\gamma}=\frac{1}{\sin\varrho''}\,\frac{\cos\delta''}{\cos r},$$

(4) $$\frac{1}{\sin\alpha}+\frac{1}{\sin\beta}-\frac{1}{\sin\gamma}=\frac{1}{\sin\varrho'''}\,\frac{\cos\delta'''}{\cos r}.$$

Equations corresponding to (5), (6) in the plane can be found in the same way. Analogies (2), (3), (4) are new to me, but it is hardly probable that so simple relations have not before been observed. — David Trowbridge, Perry City, N. Y.

VI. *Magic Table.* — The following curious and ingenious table is taken from an old newspaper, where it appears under the head of "*Magic Table to tell a Lady's Age.*"

1st.		2d.		3d.		4th.		5th.		6th.	
1		2		4		8		16		32	
1	33	2	34	4	36	8	40	16	48	32	48
3	35	3	35	5	37	9	41	17	49	33	49
5	37	6	38	6	38	10	42	18	50	34	50
7	39	7	39	7	39	11	43	19	51	35	51
9	41	10	42	12	44	12	44	20	52	36	52
11	43	11	43	13	45	13	45	21	53	37	53
13	45	14	46	14	46	14	46	22	54	38	54
15	47	15	47	15	47	15	47	23	55	39	55
17	49	18	50	20	52	24	56	24	56	40	56
19	51	19	51	21	53	25	57	25	57	41	57
21	53	22	54	22	54	26	58	26	58	42	58
23	55	23	55	23	55	27	59	27	59	43	59
25	57	26	58	28	60	28	60	28	60	44	60
27	59	27	59	29	61	29	61	29	61	45	61
29	61	30	62	30	62	30	62	30	62	46	62
31	63	31	63	31	63	31	63	31	63	47	63

The peculiarity of the table is this: If all the columns in which a particular number is found be indicated, the sum of the numbers standing at the head of such columns will equal the particular number. For instance, a particular number is found in the 1st, 2d, 4th, 5th, and 6th columns; what is the number? The sum $1 + 2 + 8 + 16 + 32 = 59$ is the required number. — Artemas Martin, Franklin, Venango County, Pa.

VII. In Robinson's Algebra is found the following problem: — "A man borrowed $60; what sum shall he pay daily to cancel the debt, principal and interest, in 60 days; interest at 10 per cent for 12 months of 30 days each?" $\$1\frac{5}{600}$ and $\$1\frac{61}{7200}$ are the answers given in different editions. Is either of them *correct*, or is the true answer $\$1\frac{61}{7259}$? — D. W. Hoyt, Professor of Mathematics, New Hampton Institute, Fairfax, Vt.

Mathematical Monthly Notices.

Annual Report of the Board of Regents of the Smithsonian Institution, showing the Operations, Expenditures, and Condition of the Institution for the Year 1861.

This Report opens, as usual, with that of PROFESSOR HENRY, — the Secretary of the Institution, — giving the history of its transactions for the year; from which we learn that the eleventh volume of the "Smithsonian Contributions to Knowledge" is nearly ready, containing the following memoirs: On North American Oölogy, by DR. THOMAS M. BREWER; On the Total Eclipse of the Sun, September 7, 1858, observed near Olmos, Peru, by LIEUT. GILLIS; Discussion of the Magnetic and Meteorological Observations made at the Girard College Observatory, Philadelphia, from 1840 to 1845, by A. D. BACHE, LL. D.; A Memoir, by DR. JOHN LE CONTE, on the Coleoptera, or Beetles, known to inhabit the Middle and Eastern portions of the great Central Regions of temperate North America; Result of Magnetical and Hypsometrical Observations in Mexico, — to which is appended Notes on the Volcano of Popocatapetl and its vicinity; On the American Storm of December 20, 1836, and the European Storm of December 25, of the same year, by PROFESSOR ELIAS LOOMIS. The General Appendix of the Report contains: Four Lectures on Agricultural Chemistry, by PROFESSOR S. W. JOHNSON; Lectures on the Shells of the Gulf of California, by P. P. CARPENTER; Latest Researches of MAEDLER relating to the General Movement of the Stars around a Central Body; Report on the Transactions of the Society of Physics and Natural History of Geneva, from July, 1858, to June, 1859; Present State of Ethnology in relation to the form of the Human Skull; Memoir of PYRAMUS DE CANDOLLE; On the Means which will be available for Correcting the Measure of the Sun's Distance in the next twenty-five years; Reports on the State of Knowledge of Radiant Heat, made to the British Association for the Advancement of Science, at the Meetings in 1832, 1840, and 1854; Description of the Magnetic Observatory at the Smithsonian Institution; On the use of the Galvanometer as a Measuring Instrument; On Observation of Earthquake Phenomena; Meteorological Instruments; On filling Barometer Tubes; Account of the Construction of a Standard Barometer, and Description of the Apparatus and Processes employed in the Verification of Barometers at the Kew Observatory.

The following problems are from "The Lady's and Gentleman's Diary" for the year 1861: —

Question I. Given the sum of the two sides, the radius of the escribed circle touching the base externally, and either of the angles at the base, to construct the triangle. W. H. LEVY, Shalbourne.

II. If the distance between the angular points and the points of contact of the escribed circles of a triangle be in harmonical progression, the escribed radii will be in arithmetical progression, and *vice versa*. — THOMAS WILKINSON, Burnley.

III. If, from the angular points of the triangle ABC, two lines be drawn, making the same constant angles with the adjacent side, two triangles $A'B'C$, $A''B''C''$ are formed similar to the original; and the circles $AA'A''$, $BB'B''$, $CC'C''$ pass through the same point. Also, if r_1, r_2, r_3 be the radii of these circles, and R that of ABC; then $r_1, r_2, r_3 = R_3$: and if O_1, O_2, O_3, O be the centres of the circles, $16 \Delta ABC \times \Delta O_1, O_2, O_3 = a^2b^2 + b^2c^2 + c^2a^2$ — EDWARD RUTTER, Sunderland.

IV. Given the perpendicular, the vertical angle and the sum of the three sides to construct the triangle. — W. H. LEVY, Shalbourne.

V. From a small plane on the floor of a room the horizontal breadth of a window in α, and the angles of elevation of its top and bottom are β and β' respectively. Show that the direct illumination of the plane by the light of the sky through the window, is to its illumination by the whole sky, supposed of uniform brightness, as $x \sin(\beta + \beta') \sin(\beta - \beta')$ to 2π. — THOMAS DOBSON, B. A., Liverpool.

VI. PC, PC', at right angles to each other, are tangents to an ellipse, and $CQ, C'Q$ the corresponding normals. Show that the locus of the intersection of CC', PQ bisects the ellipse. — STEPHEN WATSON, Haydonbridge.

VII. Let ABC be any triangle; O, O_1, O_2, O_3 the centres of its inscribed and escribed circles respectively. Then the twelve circles described about the component triangles of the complete quadrilaterals $O_1O_2OO_3$, $O_2O_1OO_3$, $O_3O_2OO_1$, will intersect, four and four, in A, B, C and their centres will lie, two and two, in six points on the circumference of the circumscribing circle. — T. T. WILKINSON, Burnley.

VIII. A circular target, for rifle-practice, weighing 18lbs., and measuring 2 feet in diameter is suspended from a horizontal axis, about which it is capable of revolving freely, the centre of the target being at a distance of 6 feet from the axis. Find at what distance from the axis it must be struck by a rifle ball, weighing 8 ounces, and moving with a given velocity, in order that the ball and target may move off together with the greatest possible velocity. — STEPHEN FENWICK, of the Royal Military Academy, Woolwich.

IX. O_2O_3 are the centres of the escribed circles, touching the sides AC, AB, of the triangle ABC, externally. Join O_2B, O_3C, cutting the sides in P_1, Q_1; O_2Q_1, O_3P_1, cutting them in P_2, Q_2, and so on; then the distance of any point in the line P_nQ_n, from BC, is equal to n times the sum of its distances from AC, AB. Also the lines $P_1Q_1, P_2Q_2, \ldots\ldots P_nQ_n$, all pass through the same fixed point. — STEPHEN WATSON, Haydonbridge.

X. If from the angles of a triangle Δ, straight lines be drawn to the opposite sides, dividing them in the proportion of m to n, where $m + n = 1$, and a triangle Δ_1 be formed having the dividing lines as sides, show that $\Delta_1 = \Delta(1 - mn)$. — PETRARCH.

XI. If a straight line be drawn parallel to a given straight line, and on the same side of it with a given circle; generally four circles may be described through any point in the parallel to touch the given straight line and to touch the circle, two externally and two internally. The ratio of a mean proportional betwixt the radii of external contact to a mean proportional betwixt the radii of internal contact, will be the same, wheresoever on the parallel the point be taken; and if the parallel be drawn through the centre of the circle, it will be a ratio of equality. Required a geometrical proof. — REV. WM. MASON, Normanton, Yorkshire.

XII. Compare the times of the small oscillations of a rod of any given length, not exceeding the earth's diameter; 1st, when it is suspended by one extremity, the other just touching the earth's surface; 2dly, when it is suspended by one end at the surface, and vibrates in a diametrical perforation through the earth. — C. H. BROOKS, Great George Street, Westminster.

XIII. A small horizontal orifice is made at the bottom of a vessel containing fluid. Determine the shape of the vessel, so that the form of the jet, at the instant the vessel is empty, shall be a straight line. — REV. S. TEBAY, Rivington.

XIV. Three points being taken at random in space as the corners of a plane triangle, determine the probability that it shall be acute. — THE EDITOR.

XV. If an ellipse be inscribed in an equilateral triangle so as to have one of its foci on the inscribed circle, then the other focus will lie on the hypocycloid of three branches, whose cusps are at the angular points of the triangle. — "CONIC," of St. John's College, Cambridge.

Editorial Items.

Mr. G. W. Hill, the author of the Prize Essay printed in this number of the Monthly, is a graduate of Rutgers College, New Brunswick, N. J., and at present a member of the Lawrence Scientific School, Cambridge, Mass., pursuing his Mathematical studies under the direction of Professor Peirce.

Sir John Herschel's Portrait. — In our next number we shall present to our readers a Steel Portrait of Sir John Herschel, engraved by H. Wright Smith, Esq., of Boston, from a recent photograph, which is pronounced an excellent likeness. The number will also contain a complete catalogue of the titles of the distinguished author's works, in all *two hundred and twenty-four*.

The following gentlemen sent solutions of Prize Problems in the October number: John A. Weinbrener, Princeton College, N. J.; Asher B. Evans, Madison University, Hamilton, N. Y.; E. O. Gibson, South New Berlin, N. Y.; J. O. Ellinwood, and D. Martin, Liberal Institute, Clinton, N. Y.; Thomas M. Sechler, E. K. Leonard, Eugene B. Willard, Marietta College, Ohio; G. A. Osborne, Jr., Lawrence Scientific School; G. W. Lewis, Middlebury College, Vt.; William Timpson, White Plains, N. Y.; A. G. Barker, Waterville College, Me.; H. Teiman, Baltimore, Md.; Joseph H. Buckley, Yale Scientific School; G. B. Hicks, Cleveland, Ohio; John F. Curtis, Piedmont Institute, Liberty, Va.

Solutions of the Problems in the November number have been received from the following gentlemen: Edwin A. Dodd, Gouverneur, N. Y.; W. D. Mershon, Princeton College, N. J.; David Trowbridge, Perry City, N. Y.; G. A. Osborne, Jr., Lawrence Scientific School; G. B. Hicks, Cleveland, Ohio; Asher B. Evans, Hamilton College, N. Y.; A. G. Barker, Waterville College, Me.; H. Teiman, Baltimore, Md.; William Woolsey Johnson, Yale College, New Haven, Ct.

Books Received. — A Higher Arithmetic: embracing the Science of Numbers, and the Art of their Application. By A. Schuyler, M. A., Principal of Seneca C. Academy, and Professor (elect) of Mathematics in Baldwin University. New York: Sheldon & Co., 115 Nassau street. 1860. — Plane Trigonometry, for the Use of Colleges and Schools. With Numerous Examples. By I. Todhunter, M. A., Fellow and Principal Mathematical Lecturer of St. John's College, Cambridge. Second edition, revised. Cambridge: MacMillan & Co. 1860. — The Claim of Leibnitz to the Invention of the Differential Calculus. By Dr. H. Sloman. Translated from the German, with Considerable Alterations and New Addenda by the Author. Cambridge: MacMillan & Co. — Dodd's Mathematical Text-books. Elementary and High School Arithmetics, 2 vols.; Elementary and High School Algebras, 2 vols.; Elements of Trigonometry, Plane and Spherical; with Applications to Heights and Distances, the Areas of Polygons, Surveying, Navigation, and the Solution of Astronomical Problems; Elements of Geometry and Mensuration. New York: Published by Pratt, Oakley, & Co.

Annual Report of the Board of Regents of the Smithsonian Institution, showing the Operations, Expenditures, and Condition of the Institution for the year 1859. Washington. 1860. — The Quarterly Journal of Pure and Applied Mathematics. No. 14. October, 1860. London: John W. Parker & Son. — Nouvelles Annales de Mathématiques. Tome Vingtième. Augmenté d'un Bulletin de Bibliographie, d'Histoire et de Biographie Mathématiques. Janvier, 1861. Paris: Mallet-Bachelier.

Worcester's Quarto Dictionary The Standard

VERY SIGNIFICANT FACTS.

The following recommendations are from some of the most distinguished American and English scholars. They are but a few from many which have been received, testifying to the superiority of Worcester's Quarto Dictionary. These testimonials are of the highest value, for they have all been given during the present year, and after an examination of this work and of that which is endeavoring to hold the position of a rival. The scholars of America and of England, with scarcely an exception, have decided in favor of Worcester. Not a single scholar, equal in authority to any one mentioned below, can be cited as giving after a comparison of the two works, the preference to Webster's Dictionary. We give the testimony: —

From C. C. FELTON, LL.D., *President of Harvard College.*

Aware of the labor and care which had been devoted to this (the department of scientific terms) as well as to other parts of the work, I felt assured that Worcester's Quarto Dictionary would more nearly meet the public wants than any other hitherto published.

My expectations have been more than fulfilled. I find it not only rich beyond example in its vocabulary, but carefully elaborate in all the details, and thoroughly trustworthy as a guide to the most correct and elegant usage of the language.

From the REV. JOSEPH BOSWORTH, D.D., F.R.S., *Professor of Anglo-Saxon, Oxford, England.*

It is the most complete and practical, the very best, as well as the cheapest English Dictionary that I know.

From GEORGE P. MARSH, LL.D.

The work of Dr. Worcester is unquestionably much superior to any other general dictionary of the language in every one of these particulars (orthography, pronunciation, definition, fulness of vocabulary, and precision and distinctness of definition).

From REV. W. WHEWELL, D.D., *Master of Trinity College, England.*

I have repeatedly consulted the Dictionary since it has been in my possession, and have seen reason to think it more complete and exact than its predecessors.

From CHARLES RICHARDSON, LL.D., *the oldest living English Lexicographer, England.*

I sincerely hope you may enjoy from your brethren, both in America and England, that tribute of honor to which you have earned so undoubted a title.

From D. R. GOODWIN, D.D., *President of Trinity College, Hartford.*

It was but a short time since that I was led to commend another dictionary as, on the whole, and with some exceptions, the best and most complete thing of the kind within my knowledge. The commendation was honestly given at the time; but now *it must be withdrawn in favor of yours.* I consider your dictionary, in orthography, pronunciation, and definitions, as superior to any of its predecessors.

From REV. W. B. SPRAGUE, D.D., *of Albany, N. Y.*

My opinion of Worcester's Quarto Dictionary, after having given it as extended an examination as my circumstances would admit, is, that there is no other dictionary in the language that compares with it for completeness, accuracy, comprehensiveness, and precision, and perhaps I ought to add, that I have arrived at this conclusion rather contrary to a preconceived opinion.

From REV. HENRY A. BOARDMAN, D D,, *of Philadelphia.*

I particularly like it (the Dictionary), 1. Because of its very comprehensive character: 2 Because it adheres to the *settled orthography* of our noble language, — discarding those innovations which, however countenanced by certain publishing-houses, have never to any extent been accepted by the scholars of our country.

From LOUIS AGASSIZ, LL.D.

It is of great importance, when the nomenclature of science is gradually creeping into common use, that an English lexicon should embrace as much of it as is consistent with the language we speak. I am truly surprised and highly delighted to find you have succeeded far beyond my expectations in making the proper selection, and combining with it a remarkable degree of accuracy. More could hardly be given except in a scientific cyclopædia.

The following lines are quoted from Harper's Magazine for September. They serve to show very truthfully the comparative value of recent and old commendations: —

"INJUSTICE. — Our attention has been called to a species of injustice of which publishers are sometimes guilty, in publishing commendations of school-books without giving the dates when they were written. Especially does this merit reproval when these commendations are old, and when it is known that the writers have subsequently commended other and later publications in the same department. It will readily be seen that this is frequently not only an act of injustice to teachers who have had the courtesy to commend a book, but that it is also a fraud upon the public."

SWAN, BREWER, & TILESTON,

131 Washington Street, Boston.

No. 117 Washington Street, Boston, October, 1860.

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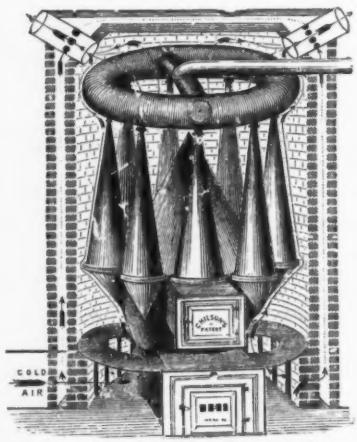

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